Part-I

1

Statical Indeterminacy

Structural analysis can be performed in two ways using :

(*a*) Force method,

(*b*) Displacement method.

For using the force method, knowledge of statical indeterminacy is necessary. It is also known as *Redundancy* of the structure. Redundant means superfluous, excess or more than necessary.

For keeping a body in equilibrium, a certain number of reactive components are necessary. This number will be equal to the number of equilibrium conditions available. For a planar structure acted upon by non-concurrent forces, the three conditions available are

(*a*) $\Sigma H = 0.$

The algebraic sum of all the forces acting in the horizontal direction should be equal to zero.

(*b*) $\Sigma V = 0.$

The algebraic sum of all the vertical forces must be equal to zero.

(*c*) $\Sigma M = 0.$

The moment of all the forces about any point in the plane should be equal to zero.

A structure in which the number of reactive components is equal to the number of available conditions is known as a *determinate structure.* In a determinate structure all the magnitudes and directions of the reactions can be found using the equilibrium conditions.

In contrast to this there are structures in which the number of reactive components will be more than the minimum that is necessary to keep the body in equilibrium. The excess number over and above the minimum is known as *Redundants.* A structure having redundants is known as *Hyper Static Structure or Redundant Structure or Statically Indeterminate System.*

Hyper means beyond. These excess unknowns cannot be determined by statics alone. Additional conditions as many in number as there are excess unknowns must be available to determine the same. These conditions will normally be available in the form of compatibility conditions and as internal releases in the system. We will discuss about these things in the coming pages.

How to Determine the Statical Indeterminacy of a Structure ?

The statical indeterminacy of a system can be assessed in three ways.

(*a*) using formula, (*b*) by axioms and (*c*) by inspection.

Three types of planar structures exist in practice :

(*a*) Flexural systems (beams and frames)

(*b*) Trusses

(*c*) Hybrid structure (combination of flexural system and Trusses).

Let us devote our attention first to flexural systems :

(*a*) *Beams.* There are three kinds of beams, *viz.*,

(*i*) Propped cantilever, (*ii*) Fixed beam and (*iii*) Continuous beams.

Beams may be subjected to vertical and inclined loadings. For beams with vertical loadings, two conditions are to be satisfied for equilibrium *viz.*,

$$\Sigma V = 0 \; ; \; \Sigma M = 0.$$

When a beam is subjected to inclined loadings, the various reactive components that exist at the various types of supports are shown in Fig. 1·1.

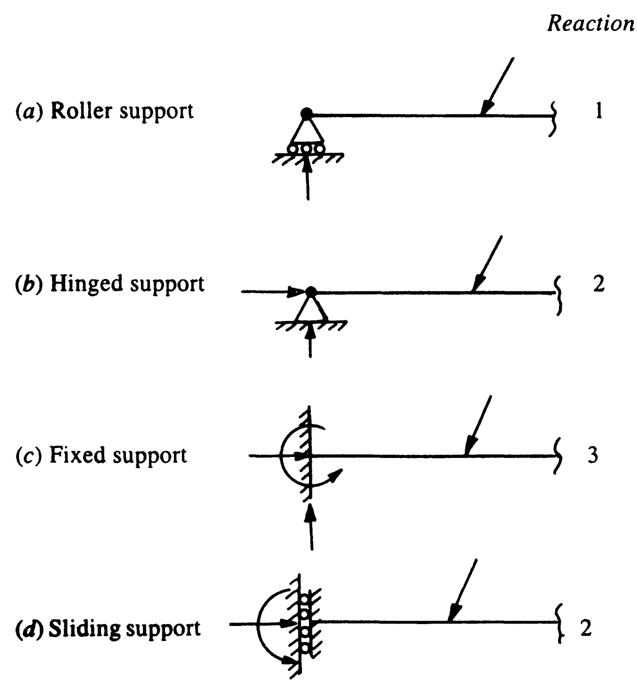


Fig. 1·1

(*a*) *Roller Support.* There will be only one reaction perpendicular to the support whatever may be the type of loading as shown in Fig. 1·1 (*a*).

(*b*) *Hinged Support.* There will be two reactions as shown in Fig. 1·1 (*b*). When the loading is vertical there will be only the vertical reaction.

(*c*) *Fixed Support.* In the case of a fixed support there will be three reactive components as shown in Fig. 1·1 (c). When the loading is vertical there will not be horizontal reaction.

(*d*) *Sliding Support.* There will be two reactive components when the loading is inclined as shown in Fig. 1·1 (*d*). When the loading is purely vertical, there will not be any horizontal reaction.

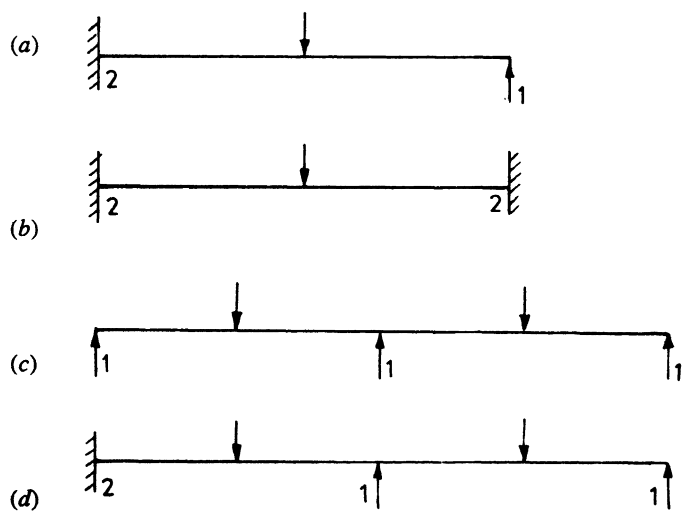
In the case of beams, there is no need to use any formula. By inspection and using the available conditions, the statical indeterminacy can be determined. When the loading is purely vertical there will be two statical conditions available, *viz.*,

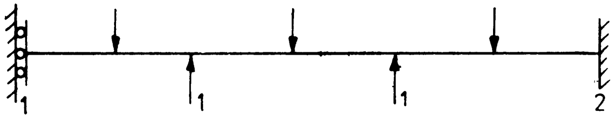
$$\sum V = 0$$

$$\sum M = 0$$

Fig. 1·2 is self-explanatory.

Beams with Internal Releases. In a beam whenever a real hinge is introduced anywhere along the length of the beam the moment at that point is zero. The beam is divided into two members by the hinge. Hinge provides one condition. In other words, if two members meet at a hinge one condition is available.



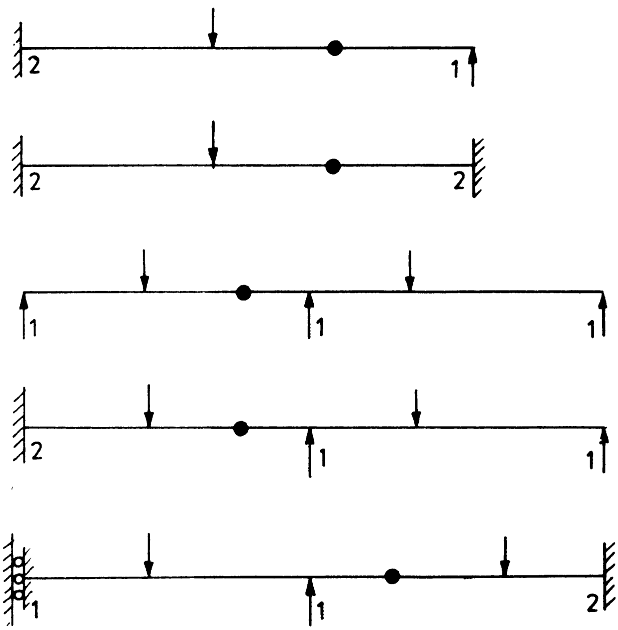
(e) 

(a) $SI = 3 - 2 = 1$, (b) $SI = 4 - 2 = 2$, (c) $SI = 3 - 2 = 1$

(d) $SI = 4 - 2 = 2$, (e) $SI = 5 - 2 = 3$.

Fig. 1·2. Statical Indeterminacy.

This information will be useful at a later time. A number of examples are given in Fig. 1·3.



(a) $SI = 3 - 2 - 1 = 0$, (b) $SI = 4 - 2 - 1 = 1$, (c) $SI = 3 - 2 - 1 = 0$

(d) $SI = 4 - 2 - 1 = 1$, (e) $SI = 4 - 2 - 1 = 1$

Fig. 1·3. Beams with internal release.

Flexural Frames

In flexural frames, the loading may be in any direction. The three equilibrium conditions available are

$$\Sigma V = 0\ ;\ \Sigma M = 0\ ;\ \Sigma H = 0.$$

A formula can be derived for finding the statical indeterminacy of frames without internal releases. The formula is as follows :

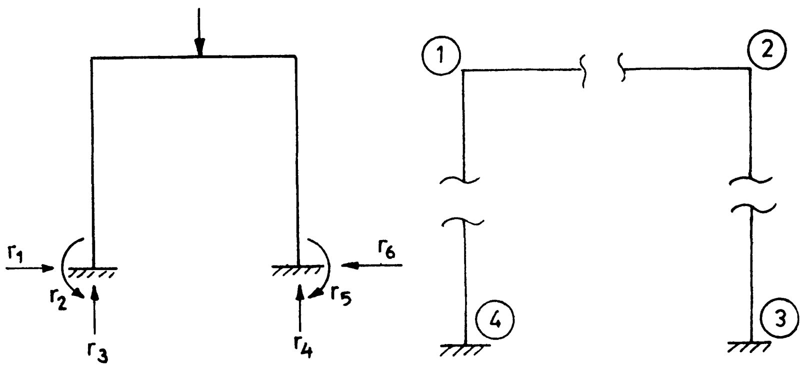
Statical Indeterminacy = $3m + r - 3j$

where m = number of members

r = total number of reactions

j = total number of joints including the supports.

Derivation of the Formula



(a) Frame with reactions. (b) Free body diagrams.

Fig. 1·4.

Fig. 1·4 (a) shows a portal frame with external reactions as r_1, r_2, r_3, r_4, r_5 and r_6. Let the total number be designated as r. The portal frame is cut into four segments with four joints as shown in Fig. 1·4 (b). Each section cuts a member. In each member there are *three* stress resultants, *i.e.*, a moment, an axial force and a shear force. Let there be m members in the frame. Each free body offers *three conditions.* Let there be j joints. In other words each joint offers three conditions. Now

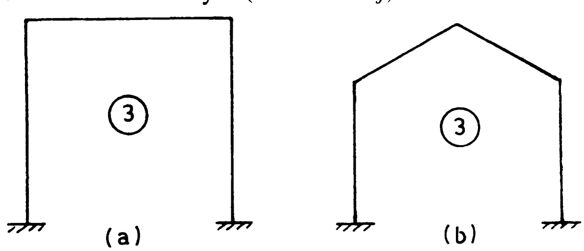
Total number of unknown stress resultants in the various members = $3m$.

Total number of reactions = r.

Total number of unknowns = $3m + r$

Conditions available = $3j$.

∴ Statical indeterminacy = $(3m + r - 3j)$.



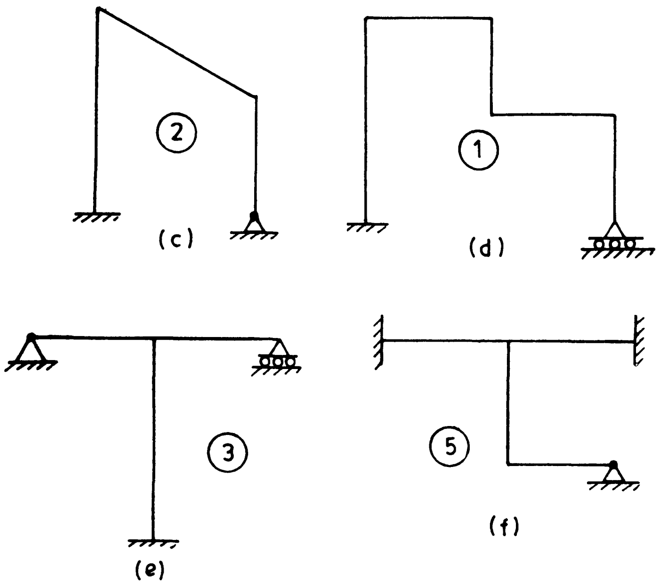


Fig. 1·5. Open frames.

This formula holds good for frames without internal releases. A number of examples will be given in Fig. 1·5 showing the application of the formula.

In the Fig. 1·5 the statical indeterminacy is shown ringed.

Fig. 1·5 (*a*) $SI = 3 \times 3 + 6 - 3 \times 4 = 3$

Fig. 1·5 (*b*) $SI = 3 \times 4 + 6 - 3 \times 5 = 3$

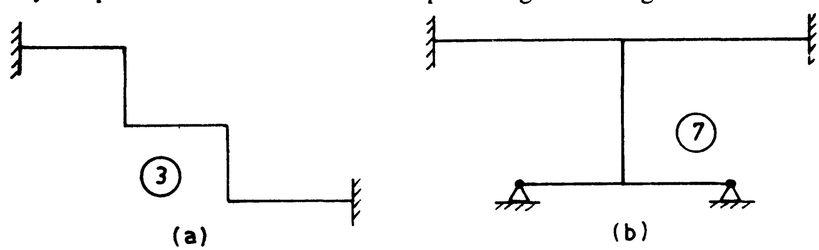
Fig. 1·5 (*c*) $SI = 3 \times 3 + 5 - 3 \times 4 = 2$

Fig. 1·5 (*d*) $SI = 3 \times 5 + 4 - 3 \times 6 = 1$

Fig. 1·5 (*e*) $SI = 3 \times 3 + 6 - 3 \times 4 = 3$

Fig. 1·5 (*f*) $SI = 3 \times 4 + 8 - 3 \times 5 = 5$. **Ans.**

These frames are known as open frames. The formula is applicable only to open frames. Some more examples are given in Fig. 1·6.



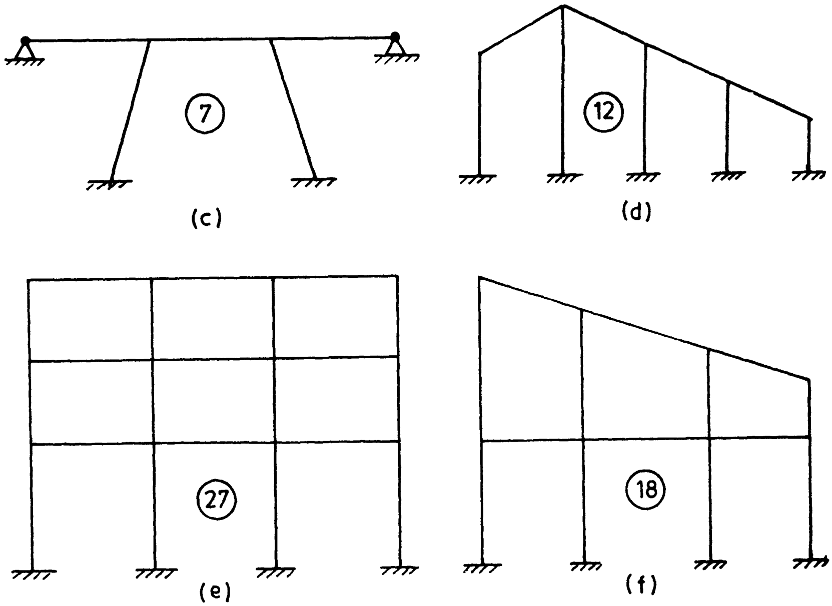


Fig. 1·6. Open frames.

Fig. 1·6 (*a*) $SI = 3 \times 5 + 6 - 3 \times 6 = 3$

Fig. 1·6 (*b*) $SI = 3 \times 5 + 10 - 3 \times 6 = 7$

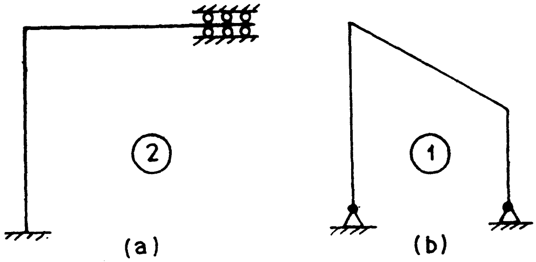
Fig. 1·6 (*c*) $SI = 3 \times 5 + 10 - 3 \times 6 = 7$

Fig. 1·6 (*d*) $SI = 3 \times 9 + 15 - 3 \times 10 = 12$

Fig. 1·6 (*e*) $SI = 3 \times 21 + 12 - 3 \times 16 = 27$

Fig. 1·6 (*f*) $SI = 3 \times 14 + 12 - 3 \times 12 = 18$ **Ans.**

Another set of open frames is shown in Fig. 1·7. By experience and practice only, we will be in a position to distinguish between open frames and close frames. Formula derived is applicable only to open frames.



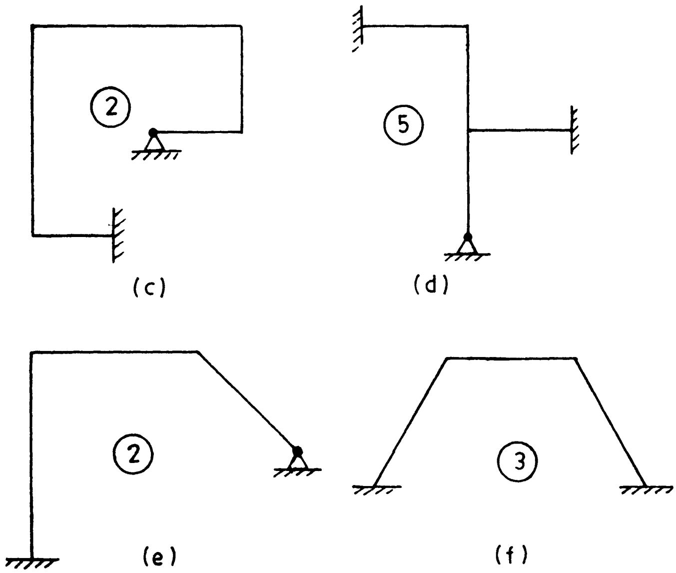
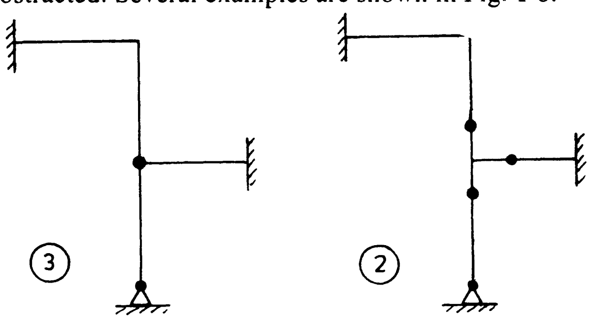


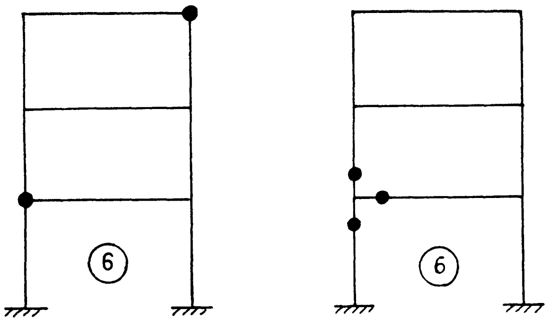
Fig. 1·7. Open frames.

Fig. 1·7 (*a*) $SI = 3 \times 2 + 5 - 3 \times 3 = 2$
Fig. 1·7 (*b*) $SI = 3 \times 3 + 4 - 3 \times 4 = 1$
Fig. 1·7 (*c*) $SI = 3 \times 5 + 5 - 3 \times 6 = 2$
Fig. 1·7 (*d*) $SI = 3 \times 4 + 8 - 3 \times 5 = 5$
Fig. 1·7 (*e*) $SI = 3 \times 3 + 5 - 3 \times 4 = 2$
Fig. 1·7 (*f*) $SI = 3 \times 3 + 6 - 3 \times 4 = 3$. **Ans.**

Frames with Internal Releases

Sometimes frames will be provided with hinges in the internal portion of the frame. We have earlier seen that a hinge connects two members. Two members meeting at a hinge provides one condition. If three members are meeting at a hinge then two conditions are available. If $(n + 1)$ members are meeting at a hinge then n conditions are available. For finding the statical indeterminacy of such a frame, the frame is analysed as if there were no internal hinges. Then the number of conditions available are suitably substracted. Several examples are shown in Fig. 1·8.





(c) (d)

Fig. 1·8. Frames with internal hinges.

The original statical indeterminacy of frames Fig. 1·8 (*a*) and Fig. (*b*) is

$$SI = 3 \times 4 + 8 - 3 \times 5 = 5.$$

Now the SI is found by subtracting suitably the conditions provided by the internal hinges.

Fig. 1·8 (*a*) $SI = 5 - 2 = 3$

Fig. 1·8 (*b*) $SI = 5 - 3 = 2$

Note that in Fig. 1·8 (*a*), internal hinge is provided exactly at the junction of three members. Therefore, such a hinge provides only two conditions whereas in Fig. 1·8 (*b*) the hinges are placed in the members. So there are three conditions available.

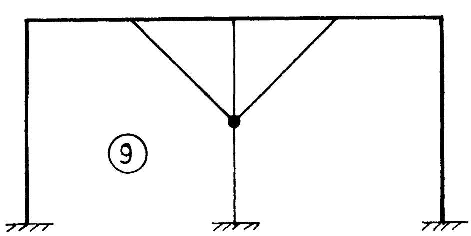
The original statical indeterminacy of frames shown in Fig. 1·8 (*c*) and Fig. 1·8 (*d*) is original

$SI = 3 \times 9 + 6 - 3 \times 8 = 9.$

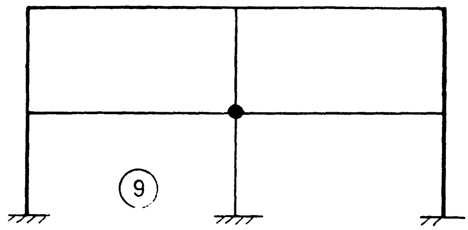
Now the conditions available are suitably subtracted.

Fig. 1·8 (*c*) $SI = 9 - (2 + 1) = 6$

Fig. 1·8 (*d*) $SI = 9 - 3 = 6$. **Ans.**



(*a*)



(*b*)

Fig. 1·9. Frames with internal hinge.

Some more examples are given in Fig. 1·9, Original SI of Fig. 1·9 (*a*) $= 3 \times 10 + 9 - 9 \times 3 = 12$.

After suitable subtraction, $SI = 12 - 3 = 9$.

Now original SI of Fig. 1·9 (*b*) $= 3 \times 10 + 9 - 3 \times 9 = 12$.

After suitable subtraction, $SI = 12 - 3 = 9$. **Ans.**

Statical Indeterminacy Through Axioms

Axiom means "statement accepted as truth without proof or argument".

In open frames we have not distinguished between internal and external redundancy. For certain configurations we can find out the internal and external indeterminacies separately and then add them together for finding the total indeterminacy. For finding the internal indeterminacy axiom will be helpful.

For a cell or loop structure whatever may be the configuration, the internal indeterminacy is equal to *three*. This is axiom. Examples are shown in Fig. 1.10.

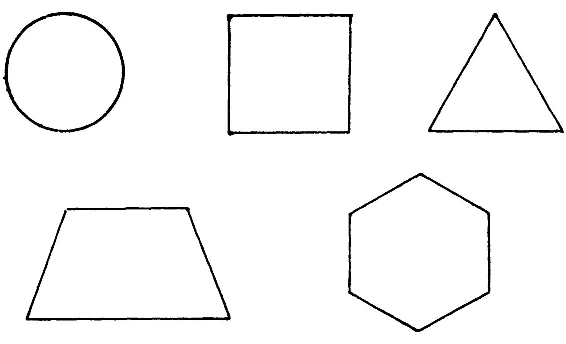


Fig. 1·10. A cell or loop or ring structure.

To make it statically determinate, all that we used is to make a cut anywhere in the system. A cut releases three redundants.

Closed Frames or Loop Structures

Examples for loop structures are box culvert, vierendeel girders, etc. In these structures internal indeterminacy is found using axiom and external indeterminacy is reckoned just like beams using the three conditions.

$$\Sigma V = 0 \; ; \; \Sigma M = 0 \; ; \; \Sigma H = 0$$

Examples are given in Fig. 1.11.

The external in determinacy of the structures shown in Fig. 1.11 is zero. These structures are externally determinate. The various reactions can be found using the three equilibrium conditions. On the other hand these three structures possess internal indeterminacy which is equal to *three*. It is found using the axiom.

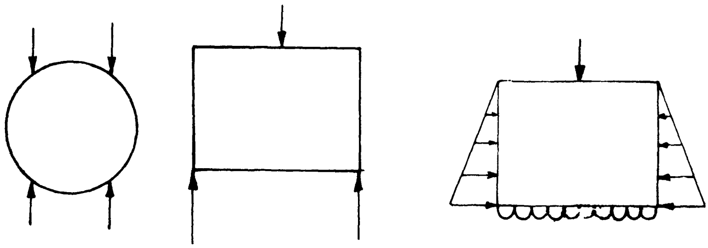
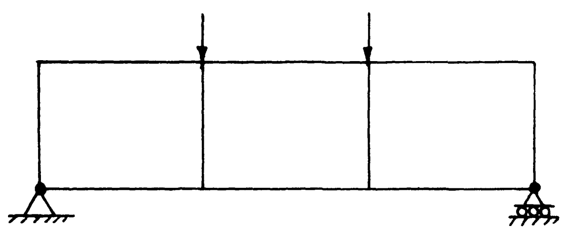


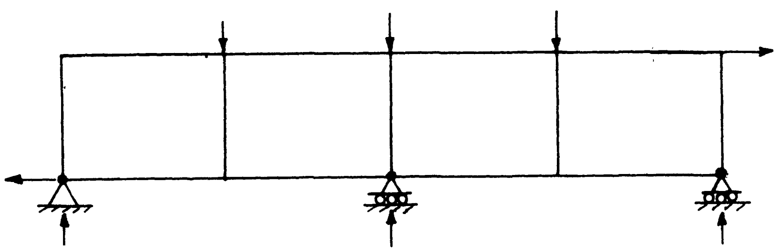
Fig. 1.11. Loop or ring structures.

Vierendeel Girder

Two Vierendeel girders are shown in Fig. 1.12.



(*a*)



(*b*)

Fig. 1.12. Vierendeel girders.

In this girder, the two external vertical reactions can be found using the two equilibrium conditions $\Sigma V = 0$, and $\Sigma M = 0$. Therefore, it is statically determinate externally. The external indeterminacy is equal to zero. On the other hand, it is internally indeterminate. There are *three* loops or rings or cells. Each cell has three degrees of statical indeterminacy. Therefore, internal indeterminacy is $3 \times 3 = 9$.

Total indeterminacy = External + Internal = $0 + 3 \times 3 = 9$ **Ans.**

Fig. 1.12 (b). In this girder, there are four external reactions and three conditions are available.

$\therefore$ External indeterminacy = $4 - 3 = 1$

There are four cells or loops or rings in this girder. Therefore, internal indeterminacy is $4 \times 3 = 12$

$\therefore$ Total indeterminacy = External + Internal = $1 + 12 = 13$ **Ans.**

With these examples we are in a position to distinguish a given structure as open frame or closed frame. In a closed frame when internal releases are provided, the same procedure indicated earlier can be adopted. It may be noted that the formula,

$$SI = 3m + r - 3j$$

is not applicable to closed frames. In Fig. 1·12 (*a*) and Fig. 1·12 (*b*), the dark shaded circle over the supports should not be considered as internal hinges.

Beam Count or Cell Count Method

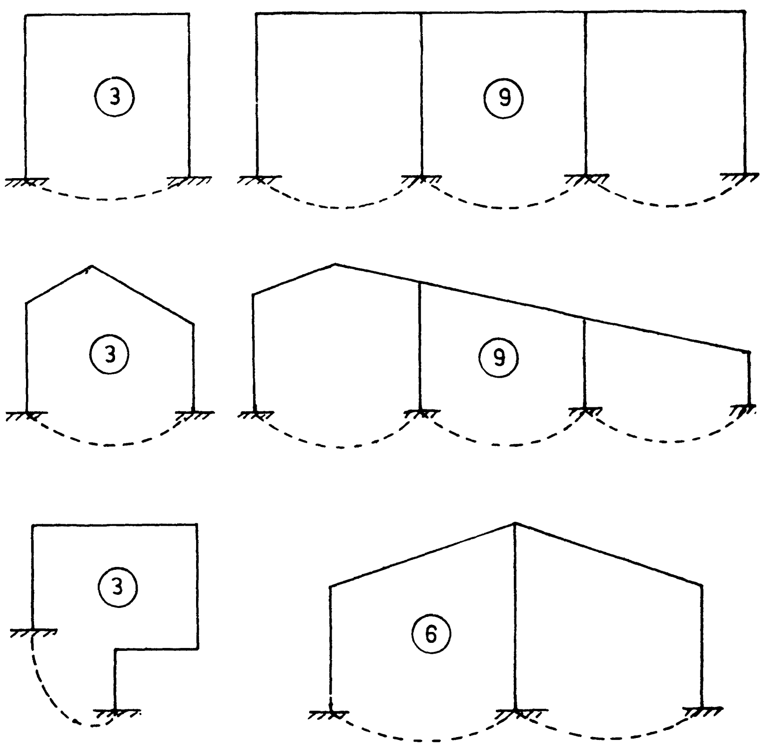
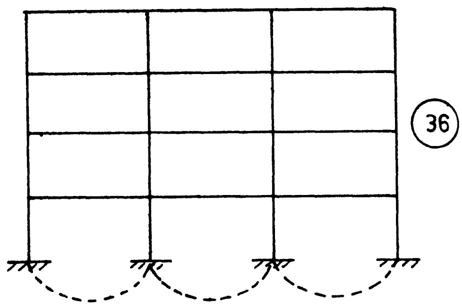


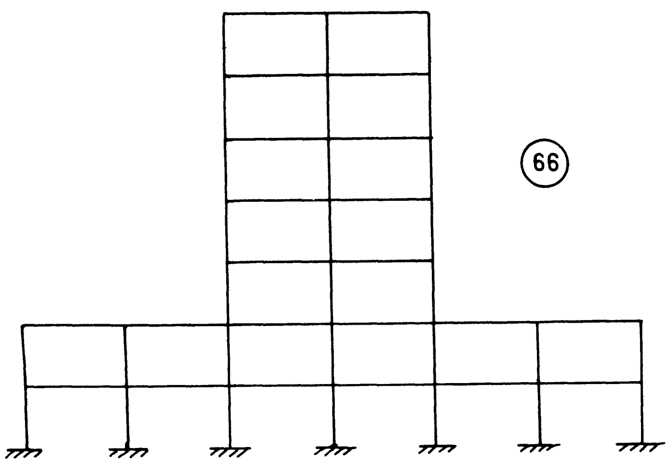
Fig. 1·13. Cell count method.

This is an easy and short cut method of finding the statical indeterminacy quickly for certain types of structures with *fixed* support conditions. The necessary condition is that the number of beams counted should be equal to the number of cells. Then the number of cells multiplied by three gives the statical indeterminacy. Fig. 1·13 gives some examples. For obtaining the cell, the fixed supports are connected by imaginary dotted lines as shown in Fig. 1·13.

The number of cells can be counted easily and upon multiplication by three gives the statical indeterminacy. The cell count method is more powerful than the beam count method which is used in the case of frames with rectilinear elements as shown in Fig. 1·14.



(*a*)



(*b*)

Fig. 1·14. Beam count method.

The number of beams are counted and checked with the number of cells. Then upon multiplication by three gives the statical indeterminacy. The statical indeterminacy is shown ringed in Fig. 1·14.

Trusses

In trusses, the statical indeterminacy is due to

1. Internal indeterminacy and 2. External indeterminacy.

Trusses can be classified as

(*a*) Single joint truss and (*b*) Multiple joint truss.

A truss member can carry either compressive or tensile force only. Each free joint furnishes two equilibrium conditions, $\sum V = 0$ and $\sum H = 0$.

(*a*) **Single free joint truss.** Statical indeterminacy = number of members meeting at the free joint − 2

i.e. $SI = n - 2$

where n = number of members meeting at the free joint.

In this structure only internal indeterminacy exists. Fig. 1·15 shows a number of examples.

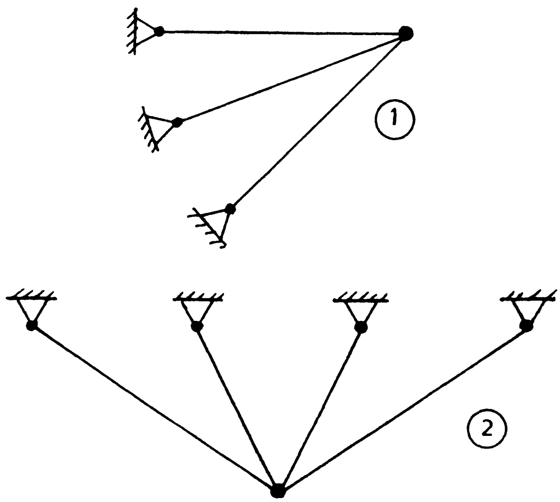


Fig. 1·15. Single free joint.

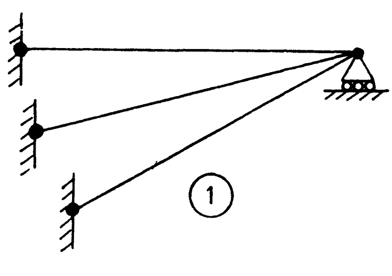


Fig. 1·16. Single joint truss with support.

(*b*) **Single joint truss with support.** This truss consists of both external and internal indeterminacy. These can be ascertained by inspection of the truss.

External $SI = 1$

Internal $SI = 1$

Total $SI = 2$ **Ans.**

Multiple Joint Frames

These frames may have either internal indeterminacy or external indeterminacy or both.

Internal indeterminacy can be known by this formula

$$n = 2j - 3$$

where n = number of members in the truss

j = number of joints.

If the left hand side and right hand side both are equal, then the truss is said to be internally determinate. If n is greater than the right side the excess number indicates the internal indeterminacy. Examples are shown in Fig. 1·17.

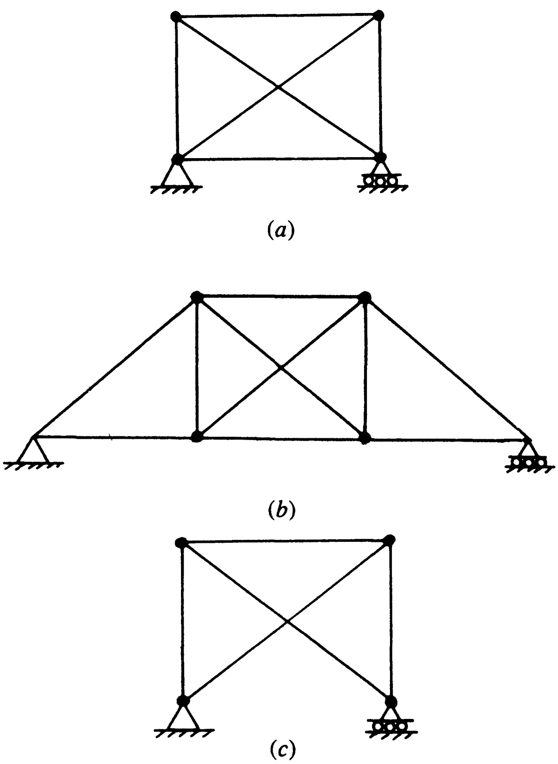


Fig. 1·17.

Fig. 1·17 (a) : $n = 2j - 3$

$$6 > (2 \times 4 - 3 = 5)$$

Excess is one. Therefore internal indeterminacy is one.

Fig. 1·17. (b) : $10 > (2 \times 6 - 3 = 9)$

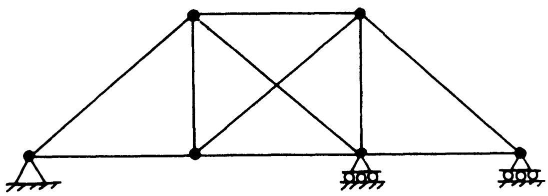
Excess is one . Internal indeterminacy is one.

Fig. 1·17. (c) : $5 = 2 \times 4 - 3 = 5$

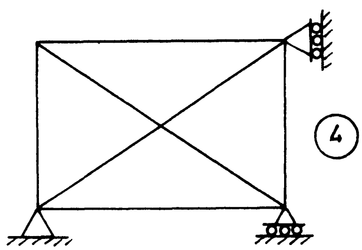
Internally determinate. Note that it has got only four joints. Intersection of diagonals is not a joint in the above three figures.

All the three trusses are externally determinate.

External indeterminacy is determined just as we did in the case of beams. Consider Fig. 1·18.



(*a*)



(*b*)

Fig. 1·18.

In Fig. 1·18 both the figures we have internal and external indeterminacy as equal to one. Therefore total indeterminacy of each system is equal to two.

Hybrid Structures

Sometimes structures may have in them both trusses and flexural frames or beams connected together. Such structures should be examined carefully and the statical indeterminacy is determined.

Example 1. The cantilever and the truss are connected by a tension rod. The cantilever and the truss are both statically determinate. Hence the only unknown force is in the rod. Therefore, statical indeterminacy of the structure is one.

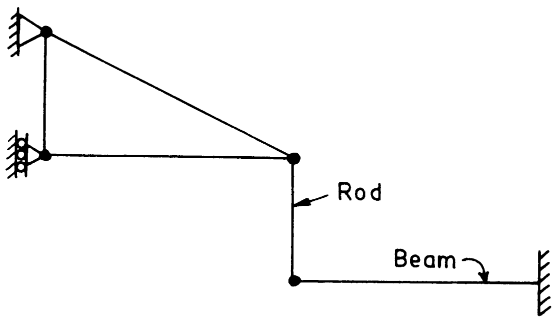


Fig. 1·19. Hybrid structure.

Example 2. The cantilever is held by two wires. The cantilever by itself is statically determinate. The forces in the two wires are unknowns. The statical indeterminacy of the system is two.

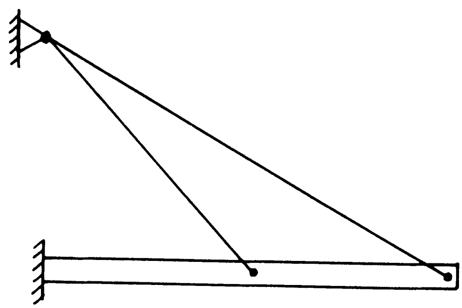


Fig. 1·20. Cantilever beams.

Example 3. The rigid bar can be maintained is equilibrium by one wire. Therefore, the force in the second wire is redundant. The system possesses one degree of statical indeterminacy.

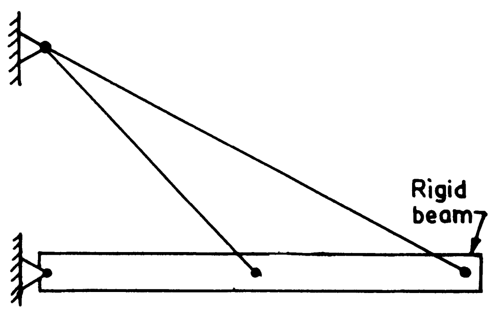


Fig. 1·21. Rigid beam.

Statical Indeterminacy of Three Dimensional Flexural Frames

In a three dimensional flexural frame, each member will have six stress resultants, *i.e.,* three moments and three forces. A cut in a member releases six stress resultants. In other words, a cut nullifies or removes six stress resultants. For finding the statical indeterminacy of such a frame, there is no single formula available covering all the cases. However, a formula can be derived for a frame with fixed base and without hinges. This formula may be modified to account for frames having hinged column bases.

(*a*) Formula for Frames with Fixed Columns :

Let us consider a three dimensional fixed frame. Let there be 'm' members, 'r' reactive components and 'j' joints including fixed bases. Each joint of the frame furnishes six equilibrium conditions, *viz.,* (namely)

$$\sum F_x = 0,\ \sum F_y = 0,\ \sum F_z = 0,\ \sum M_x = 0,\ \sum M_y = 0 \text{ and } \sum M_z = 0$$

Number of unknowns in members $= 6\,m$

Total no. of reactions $= r$

Available joint conditions $= 6\,j$

Statical Indeterminacy = No of unknowns – Available Conditions.

$$\boxed{\text{S.I.} = 6m + r - 6j}$$

Besides this formula, two other short-cut methods exist for determining the SI even though they are less powerful than the above formula.

(*i*) **Beam Cut Method :** In this method, all the beams are cut. Each cut removes six stress resultants. When the beams are cut, each cut portion along with the column becomes statically determinate. (A tree is produced by these cuts). The structure becomes determinate. Hence

$$\boxed{\text{Statically Indeterminacy} = \text{No. of beams} \times 6}$$

There is limitation for this formula which we shall indicate at the time of illustrating the solution for that particular problem.

(*ii*) **Cell or Loop or Ring Count Method :** A cell in a three dimensional structure has six stress resultants. In the given system all the *vertical* cells are counted ignoring horizontal cells since the members in the horizontal cells have already been considered in the vertical cell. Then

$$\boxed{\text{Statically Indeterminacy} = \text{No. of vertical cells} \times 6}$$

This method also has limitation which we shall bring out while doing problems.

Solved Examples

Example 1. *Compute the statical Indeterminacy of the three dimensional frame shown in Fig. 1·22.*

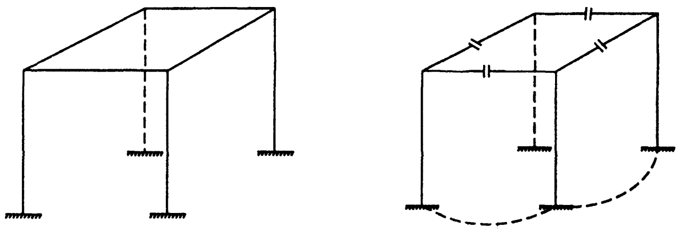


Fig. 1·22 (*a*) Fig. 1·22 (*b*)

Sol.

(*i*) *By Formula* : $m = 8, r = 24, j = 8$

$$SI = 6m + r - 6j = 6 \times 8 + 24 - 6 \times 8 = \mathbf{24} \text{ Ans.}$$

(*ii*) *By Beam Count* : There are 4 beams in the structure. Therefore

$$SI = 6n = 6 \times 4 = \mathbf{24} \text{ Ans.}$$

(*iii*) *By Vertical Cell Count* : There are four vertical cells. So

$$SI = 6 \times 4 = \mathbf{24} \text{ Ans.}$$

Example 2. *Find the statical indeterminacy of the three dimensional frame shown in Fig. 1·23.*

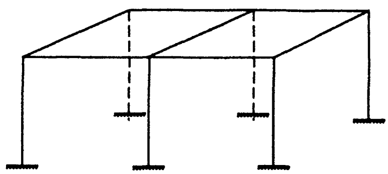


Fig. 1·23.

Sol.

(*i*) *By Formula* : $m = 13, r = 36, j = 12$

$$SI = 6 \times 13 + 36 - 6 \times 12 = 78 + 36 - 72 = \mathbf{42}$$

Ans.

(*ii*) *By Beam Count* : There are 7 beams in the structure. So

$$SI = 6n = 6 \times 7 = \mathbf{42} \text{ Ans.}$$

(*iii*) *By Cell Count* : There are 7 vertical cells. So

$$SI = 6 \times 7 = \mathbf{42} \text{ Ans.}$$

Example 3. *Compute the statical indeterminacy of the given system shown in Fig. 1·24.*

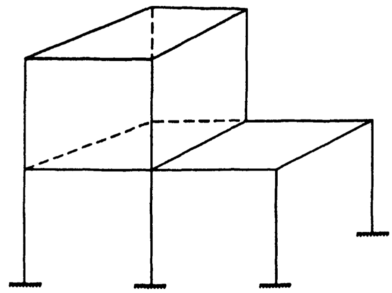


Fig. 1·24.

Sol.

(*i*) *By Formula* : $m = 21$, $r = 36$ and $j = 16$

$$SI = 6 \times 21 + 36 - 96 = \mathbf{66} \quad \textbf{Ans.}$$

(*ii*) *By Beam Count* : There are 11 beams. Hence $n = 11$

$$SI = 6\,n = 6 \times 11 = \mathbf{66} \quad \textbf{Ans.}$$

(*iii*) *By Cell Count* : There are 11 vertical cells. Hence,

$$SI = 6 \times 11 = \mathbf{66} \ \textbf{Ans.}$$

Example 4. *Two plaza frames shown in Fig. 1·25 are connected by cross beams and slabs having single bay perpendicular to the paper. Ignoring the flexural rigidity of the slabs. Compute the Statical Indeterminacy of the three dimensional frame.*

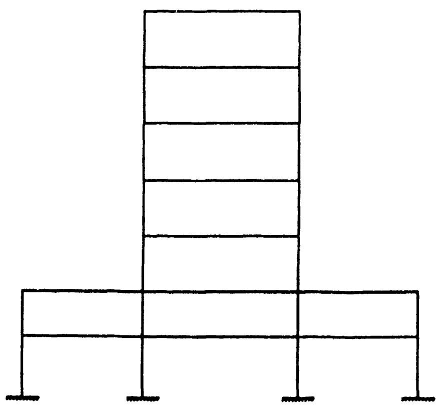


Fig. 1.25

Sol.

(*i*) *By the Formula* : $SI = 6\,m + r - 6\,j$

$$m = (29 + 29) + 18 = 76,\ r = (4 + 4) \times 6 = 48,\ J = 44$$

$$SI = (6 \times 76) + (8 \times 6) - (44 \times 6) = 456 + 48 - 264 = \mathbf{240}$$ **Ans.**

(*ii*) *By Beam Count* : There are 40 beams in the structure

$$SI = 6 \times 40 = \mathbf{240}$$ **Ans.**

(*iii*) *By Cell Count* : There are 40 vertical cells. Hence

$$SI = 6 \times 40 = \mathbf{240}$$ **Ans.**

Example 5. *Compute the statical indeterminacy of the Stepped three dimensional frame shown in Fig. 1·26.*

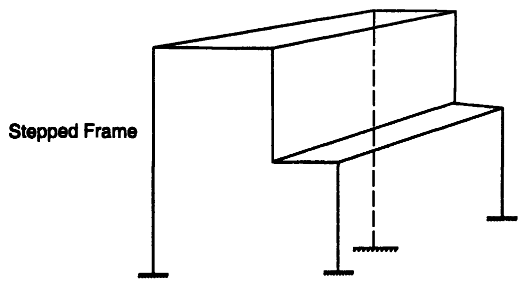


Fig. 1·26.

Sol. : $SI = 6m + r - 6j$

$m = 14$, $r = 24$ and $j = 12$

Hence $SI = (6 \times 14) + 24 - (6 \times 12) = 84 + 24 - 72 = \mathbf{36}$ **Ans.**

For stepped frames SI = cannot be found by Beam Count and Cell Count method. However, by inspection we can determine SI. The structure can be rendered statically determinate by four cuts in the four cross beams (one in each cross beam) and by removing the two fixed bases on the right extreme columns. Therefore,

$$SI = (4 \times 6) + (2 \times 6) = \mathbf{36}$$ **Ans.**

Example 6. *One of the columns in Example is hinged at the bottom. Compute the statical indeterminacy.*

Sol. For a hinged support, the reactive components are only 3. So 'r' is to be modified. Therefore in the formula $r = 6 + 6 + 6 + 3 = 21$ is to be used instead of 24 as done in the Example 1.

$$SI = 6 \times 8 + 21 - 6 \times 8 = 48 + 21 - 48 = 21$$ **Ans.**

In other words, the problem can be done considering all the supports as fixed and from the result suitable number may be subtracted. In this case this number is 3.

Example 7. Compute the static indeterminacy of the frames shown in Fig. 1·27.

Sol. (*a*) Fig. 1·27 (*a*)

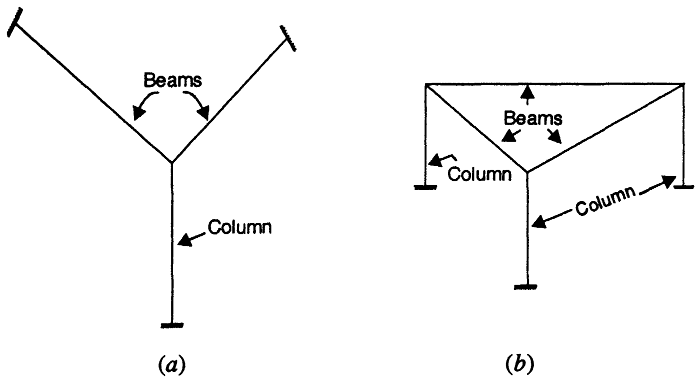


Fig. 1·27.

(*i*) *By the Formula* : $SI = 6\,m + r - 6\,j$

$= (6 \times 3) + 18 - (6 \times 4) = 36 - 24 =$ **12 Ans.**

(*ii*) *Beam Count* : $SI = 6\,n = 6 \times 2 =$ **12 Ans.**

(*iii*) *Cell Count* : $SI = 6 \times 2 =$ **12 Ans.**

(*b*) Fig. 1·27 (*b*)

(*i*) *By the Formula* : $SI = 6\,m + r - 6\,j = (6 \times 6) + 18 - (6 \times 6) =$ **18 Ans.**

(*ii*) *Beam Count* : $SI = 6\,n = 6 \times 3 =$ **18 Ans.**

(*iii*) *Cell Count* : $SI = 6 \times 3 =$ **18 Ans.**

By making cuts in the beams, the structures can be made statically determinate.